Free Vibrations of a Thin, Stiffened, Cylindrical Shallow Shell

Zahit Mecitoğlu* and M. Cengiz Dökmeci†
Istanbul Technical University, Maslak, 80626
İstanbul, Turkey

Introduction

STIFFENED plates and shells have found wide applications in aircraft, spacecraft, and ships. Hence, the dynamic characteristics of stiffened plates[1,2] and shells[3-6] have been studied extensively. The purpose of the present work is to analyze the free vibrations of a stiffened shallow shell numerically by the collocation method within the frame of the theory of classical thin orthotropic shallow shells. The eccentricity of stiffeners is included through the moments of inertia of the stiffeners. The vibration characteristics of unidirectionally and orthogonally stiffened shallow shells are studied for various geometrical and material parameters.

Governing Equations

The governing equations for the vibrations of stiffened shallow shells depicted in Fig. 1 may be expressed in the following coupled form[7]

$$B_x\frac{\partial^4 w}{\partial x^4} + 2H\frac{\partial^4 w}{\partial x^2 \partial y^2} + B_y\frac{\partial^4 w}{\partial y^4} + \frac{1}{R}\frac{\partial^2 \psi}{\partial x^2} + \rho h_e \frac{\partial^2 w}{\partial t^2} = 0 \qquad \text{(1a)}$$

$$D_x\frac{\partial^4 \psi}{\partial x^4} + 2T\frac{\partial^4 \psi}{\partial x^2 \partial y^2} + D_y\frac{\partial^4 \psi}{\partial y^4} - \frac{Eh}{R}\frac{\partial^2 \psi}{\partial x^2} = 0 \qquad \text{(1b)}$$

where w and ψ are the displacement components of the middle surface of the shell in the z direction and the stress function, respectively, and E, ρ, and h_e denote Young's modulus, mass density per unit volume, and equivalent shell thickness, respectively. The coefficients in Eqs. (1a) and (1b) are defined by

$$B_x = D + (EI/s)_x, \qquad D_x = 1/(1-\nu^2) + (EA)_x/h$$

$$B_y = D + (EI/s)_y, \qquad D_y = 1/(1-\nu^2) + (EA)_y/h$$

$$H = D + [(GJ/s)_x + (GJ/s)_y]$$

$$T = (1-\nu^2)[D_x D_y - \nu/(1-\nu^2)^2] \qquad \text{(2)}$$

Here, I, J, and A are the moment of inertia, the torsional rigidity, and the cross-sectional area of a stiffener, respectively; D is the flexural rigidity of shallow shell; and G and ν are the shear modulus and Poisson's ratio, respectively.

Eliminating the coupling between the two equations, and assuming the harmonic motion, $w(x,y,t) = W(x,y)e^{i\omega t}$, the governing equations (1) take the form

$$B_xD_x\frac{\partial^8 W}{\partial x^8} + 2(HD_x + TB_x)\frac{\partial^8 W}{\partial x^6 \partial y^2} + (D_xB_y + 4TH + D_yB_x)\frac{\partial^8 W}{\partial x^4 \partial y^4} + 2(TB_y + HD_y)\frac{\partial^8 W}{\partial x^2 \partial y^6} + B_yD_y\frac{\partial^8 W}{\partial y^8} + \frac{Eh}{R^2}\frac{\partial^4 W}{\partial x^4} - \rho h_e \omega^2\left(D_x\frac{\partial^4 W}{\partial x^4} + 2T\frac{\partial^4 W}{\partial x^2 \partial y^2} + D_y\frac{\partial^4 W}{\partial y^4}\right) = 0 \qquad \text{(3)}$$

for the free vibrations of the stiffened shallow shell. Here, ω is the frequency in rad/s. The boundary conditions are taken to be

$$W = \frac{\partial^2 W}{\partial x^2} = 0, \qquad x = 0, a$$

$$W = \frac{\partial^2 W}{\partial y^2} = 0, \qquad y = 0, b \qquad \text{(4)}$$

for the shallow shell with simple supports at all edges.

Equation (3) is shown to have a unique solution under the boundary conditions of Eq. (4).[8]

Method of Solution

In this study, the collocation method is used to solve the governing equation, Eq. (3), together with the boundary conditions, Eq. (4). For the simply supported boundary conditions, the displacement function is taken to be

$$W(\xi,\eta) = \sum_{m=1}^{M}\sum_{n=1}^{N} A_{mn}\sin(m\pi\xi)\sin(n\pi\eta) \qquad \text{(5)}$$

where $\xi = x/a$ and $\eta = y/b$. When this function is substituted into the governing differential equation and a frequency parameter as $\Omega = \omega a^2\sqrt{\rho h_e/D}$ is defined, we have the error function as follows

$$\epsilon = \sum_{m=1}^{M}\sum_{n=1}^{N} A_{mn}\sin(m\pi\xi)\sin(n\pi\eta)\{\pi^4[B_xD_xm^8 + 2(HD_x + TB_x)m^6n^2\lambda^2 + (D_xB_y + 4TH + D_yB_x)m^4n^4\lambda^4 + 2(TB_y + HD_y)m^2n^6\lambda^6 + B_yD_yn^8\lambda^8 + Eha^4m^4\lambda^4/R^2] - \Omega^2D[D_xm^4 + 2Tm^2n^2\lambda^2 + D_yn^4\lambda^4]\} \qquad \text{(6)}$$

where $\lambda = a/b$, the aspect ratio of shallow shell. To satisfy the governing equation at $M \times N$ specified points, one sets $\epsilon(\xi,\eta) = 0$ at these points. Then, Eq. (6) is reduced to

$$[U - \Omega^2 V]\{A_{mn}\}^T = 0 \qquad \text{(7)}$$

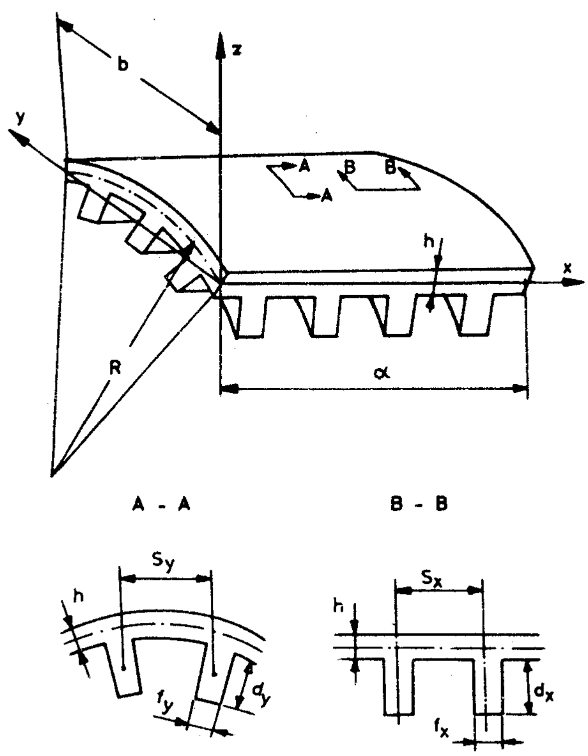

Fig. 1 Geometry of stiffened shell.

Received July 12, 1990; revision received Sept. 20, 1990; accepted for publication Nov. 29, 1990. Copyright © 1991 by the American Institute of Aeronautics and Astronautics, Inc. All rights reserved.

*Assistant Professor, Department of Aeronautical Engineering.
†Dr.-Ing. and Ph.D., Department of Naval Architecture.

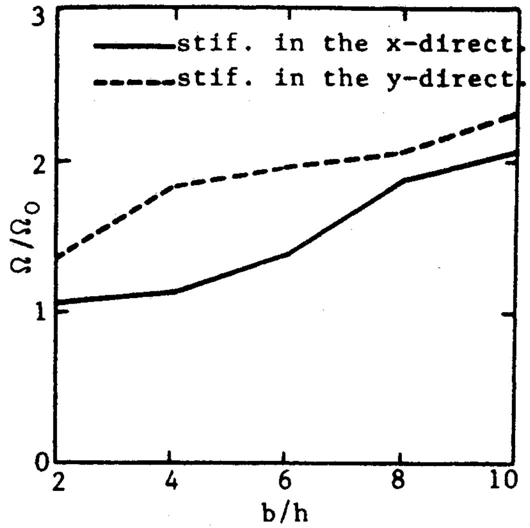

Fig. 2 Depth ratio effects on the least frequency ratio of a shallow shell: $f_x/h = 1$, $N_x = 10$, $f_y/h = 1$, and $N_y = 10$.

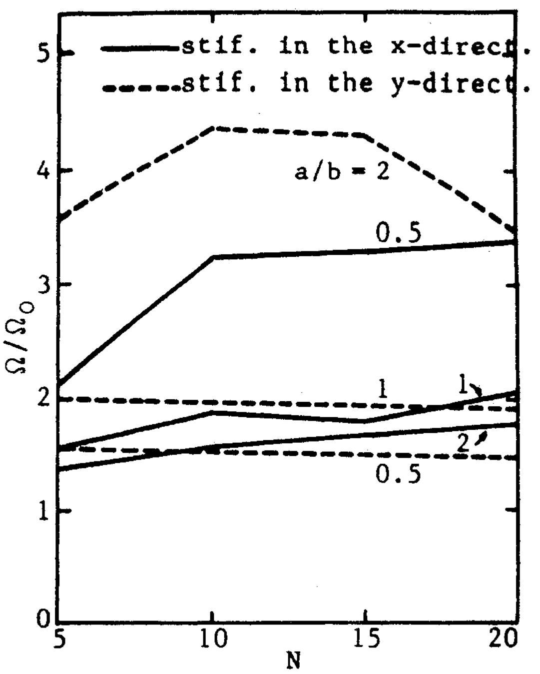

Fig. 3 Number of stiffeners effects on the least frequency ratio: $d_x/h = 8$, $f_x/h = 1$, and $f_y/h = 1$.

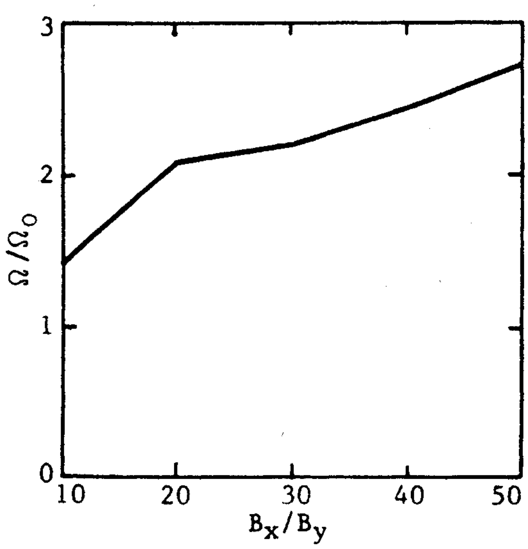

Fig. 4 Least frequency ratio vs the orthotropicity ratio.

Table 1 Frequency parameters $\Omega = \omega a^2 \sqrt{\rho h/D}$ of a stiffened plate; $a/b = 1$, $b/h = 288$

Mode	Present solution[a] NCP = 25	Present solution[a] NCP = 40	Wah[1]	Mukherjee and Mukhopadhyay[2]
1	24.044	24.044	23.904	23.906
2	49.880	50.689	49.536	47.808
3	75.226	75.216	74.736	75.024
4	96.165	95.841	95.760	96.480

[a]NCP: Number of collocation points.

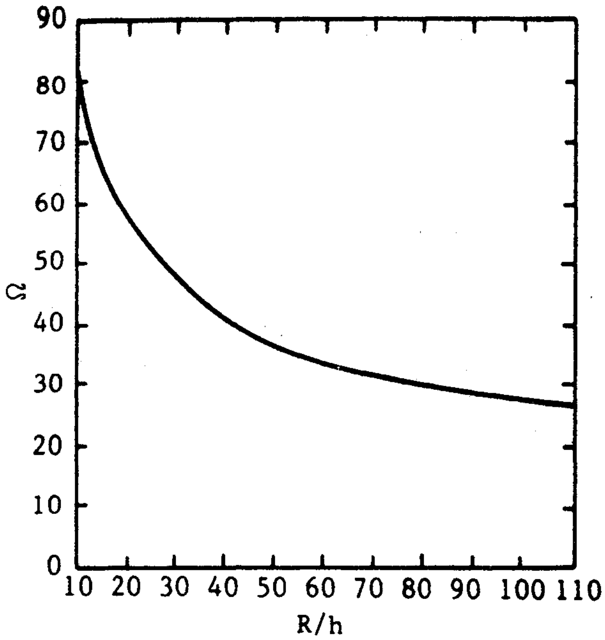

Fig. 5 Least frequency vs the curvature ratio, $N_x = N_y = 20$.

which yields a total of $M \times N$ simultaneous, linear equations to determine the frequencies and unknown parameters A_{mn}. The mode shape corresponding to each frequency can be obtained by substituting the A_{mn} parameters back into Eq. (7).

Numerical Results

The vibration characteristics are determined for stiffened shallow shells having simply supported edges, which have integral stiffeners in one or two directions. Poisson's ratio is taken to be 0.3 in all numerical examples.

The natural frequency parameters are listed in Table 1 for a stiffened plate, as a special case of the present problem. The points of collocation are selected equally spaced. As shown from Table 1, the collocation results are in good agreement wtih earlier results.[1,2]

The variation of the least frequency parameter ratio Ω/Ω_0 with the depth ratio of a stiffener is shown in Fig. 2 for the shallow shell stiffened in the x direction and for that in the y direction. Here, Ω_0 is the least frequency parameter of the unstiffened shallow shell. The geometrical properties of the stiffened shallow shell are chosen as $a/b = 1$, $b/h = 200$, and $b/R = 0.4$.

Similarly, the effect of the width ratio of a stiffener on the least frequency is investigated for the same stiffened shallow models but no practical change is found.

The variation of the least frequency with the number of stiffeners in the x and y directions is shown in Fig. 3 for shallow shells with different aspect ratios. The geometrical properties of the shallow shells are 1) $a/b = 0.5$, $b/h = 200$, and $b/R = 0.4$; 2) $a/b = 1$, $b/h = 200$ and $b/R = 0.4$; 3) $a/b = 2$, $b/h = 100$, and $b/R = 0.2$.

Finally, the variation of the lowest frequency ratio with the orthotropicity ratio B_x/B_y for the orthogonally stiffened shallow shell of square planform is shown in Fig. 4. The variation of the lowest frequency with the curvature ratio of shallow shell with square planform is shown in Fig. 5. The depth ratio and width ratio of the stiffeners are the same as the previous examples.

Conclusions

The free vibrations of a stiffened shallow shell are studied numerically by use of the collocation method on the basis of the theory of thin orthotropic shallow shells. A comparison of the present results with earlier results[1,2] shows good agreement in the case of the stiffened plate.

For a unidirectionally stiffened shallow shell, the lowest frequency increases considerably with the depth ratio of the stiffener. The influence of the width ratio on the lowest frequency is negligible expectedly. The number of the stiffeners affects the dynamic behavior of the stiffened shallow shell only for certain aspect ratios. The lowest frequency increases considerably for certain range of the orthotropicity ratio. When increasing the curvature thickness ratio, the lowest frequency decreases sharply and approaches to a limit value.

References

[1]Wah, T., "Vibration of Stiffened Plates," *Aeronautical Quarterly,* Vol. XV, Aug. 1964, pp. 285–298.

[2]Mukherjee, A., and Mukhopadhyay, M., "Finite-Element Free Vibration Analysis of Stiffened Plates," *Aeronautical Journal,* Vol. 90, Aug./Sept. 1986, pp. 267–273.

[3]Egle, D. M., and Sewall, J. L., "An Analysis of Free Vibration of Orthogonally Stiffened Cylindrical Shells with Stiffeners Treated as Discrete Elements," *AIAA Journal,* Vol. 6, No. 3, 1968, pp. 518–526.

[4]Mead, D. J., and Bardell, N. S., "Free Vibration of a Thin Cylindrical Shell with Discrete Axial Stiffeners," *Journal of Sound and Vibration,* Vol. 111, No. 2, 1986, pp. 229–250.

[5]Mecitoğlu, Z., "Free Vibrations of Stiffened Circular Cylindrical Shells," Ph.D. Dissertation, Istanbul Technical Univ., Istanbul, Turkey, 1988; see also *Journal of the Acoustical Society of America,* Vol. 85, June 1989, p. 2682.

[6]Mecitoğlu, Z., and Dökmeci, M. C., "Forced Vibrations of Stiffened Shallow Panels," *117th Meeting of Acoustical Society of America,* Syracuse Univ., New York, May 1989; see also *Journal of the Acoustical Society of America Supplemental 1,* Vol. 85, 1989, p. S118.

[7]Leissa, A. W., "Vibration of Shells," NASA SP-288, 1973.

[8]Mecitoğlu, Z., and Dökmeci, M. C., "Vibration Analysis of Stiffened Cylindrical Thin Shells," *Proceedings of the 17th Congress of the International Council of the Aeronautical Sciences,* Vol. 1, AIAA, Washington, DC, 1990, pp. 986–993.

Modal Sensitivities for Repeated Eigenvalues and Eigenvalue Derivatives

Jinsiang Shaw* and Suhada Jayasuriya†
Texas A&M University, College Station, Texas 77843

I. Introduction

RESEARCH on the modal sensitivites, i.e., the determination of derivatives of eigenvalues and eigenvectors with respect to system parameters, has been studied for quite some time, motivated mainly by its important applications in areas such as system design, system identification, and optimization. An excellent survey paper by Adelman and Haftka[1] summarizes the progress made in the study of modal sensitivities.

Received Aug. 10, 1990; revision received Dec. 1, 1990; accepted for publication Dec. 7, 1990. Copyright © 1991 by the American Institute of Aeronautics and Astronautics, Inc. All rights reserved.

*Research Associate, Mechanical Engineering Department.
†Associate Professor, Mechanical Engineering Department.

The first computationally efficient algorithm for computing the derivatives of eigenvalues and eigenvectors of real eigensystems with nonrepeated eigenvalues was introduced by Nelson[2] in 1976. It preserves the symmetry and bandedness of the original eigensystem and requires only those eigendata that are to be differentiated. Many real systems, though, exhibit repeated eigenvalues or identical frequencies and have different mode shapes. Nelson's algorithm cannot be directly applied to such systems. Ojalvo[3] extended Nelson's algorithm for solving such singularity problems arising in real symmetric systems, which was later completed independently by Mills-Curran[4] and Dailey.[5] The extended Nelson's algorithm, however, is valid only when the eigenvalue derivatives are assumed to be distinct. In fact, it will be shown that Dailey's argument for the case when both repeated eigenvalues and repeated eigenvalue derivatives are present is not correct.

This Note extends the method of Refs. 3–5 for computing the derivatives of eigenvalues and eigenvectors to include the case when both repeated eigenvalues and repeated eigenvalue derivatives are present. A symmetrical system subjected to parameter perturbations (e.g., a 4×4 cyclic matrix representing a rotor of four blades with the natural frequencies of two of its adjacent blades subjected to a small perturbation) is such a case. The present extension is based on utilizing the third derivative of the eigenvalue problem.

II. Theoretical Background

Consider the eigenvalue problem

$$(K - \lambda M)x = 0 \tag{1}$$

$$x^T M x = 1 \tag{2}$$

where M and K are $n \times n$ real symmetric matrices whose elements depend continuously on system parameters, λ is an eigenvalue, and x is the corresponding eigenvector. Moreover, it is assumed that the eigenvalue problem has m repeated eigenvalues, λ_i, $i = 1,2,\ldots,m$ when a system parameter $p = p_0$.

The objective is to determine the derivatives of the repeated eigenvalues and the derivatives of the corresponding eigenvectors, at $p = p_0$. Let $\lambda_1 = \lambda_2 = \ldots = \lambda_m = \lambda$ denote the m repeated eigenvalues, and $X \in R^{n\times m}$ be the m arbitrary eigenvectors associated with λ at $p = p_0$. The eigenvalue problem corresponding to the case of repeated eigenvalues can be written as

$$KX = MX\Lambda \tag{3}$$

$$X^T MX = I \tag{4}$$

with $\Lambda = \lambda I \in R^{m\times m}$. Consequently, the derivatives of the eigenvalues and the eigenvectors with respect to the system parameter p, at p_0, may be obtained by taking the derivatives of Eqs. (3) and (4) with respect to p. However, the derivatives of the eigenvectors may not exist since it is possible that X may not be continuous and differentiable in p because of the nonuniqueness of X. Therefore, a correct set of eigenvectors (denoted by Z), which are continuous and differentiable in p associated with λ, is needed in Eqs. (3) and (4) in order for the eigenvector derivatives to exist.

Because Z and X are both matrices of eigenvectors, there exists a transformation A such that $Z = XA$ where A is orthogonal (i.e., $A^T A = I$) due to the orthonormal constraint of the eigenvectors. References 4 and 5 give an algorithm, which is briefly described below, for computing A, Λ', and Z' where ($'$) denotes the derivative with respect to p.

Taking the derivative of the eigenvalue equation $KZ = MZ\Lambda$ yields

$$(K - \lambda M)Z' = (\lambda M' - K')Z + MZ\Lambda' \tag{5}$$